

NAME: SOL'N

### Test Prep 1

The test prep includes the two problems below. If you finish those, try the ones on the back (if you miss a point on the front, I will give you the point back if you have a question correct on the back). If you get everything (front and back) correct, I'll give you 11 out of 10 for this test prep. You are only getting 10 minutes!

1. Evaluate  $\int x e^{3x} dx$  (I encourage you to check your final answer by differentiating).

$$= \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} dx$$
$$\boxed{-\frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C}$$

$$u = x \quad dv = e^{3x} dx$$
$$du = dx \quad v = \frac{1}{3} e^{3x}$$

2. Find the numbers  $A$ ,  $B$ , and  $C$  that make the following true for all valid values of  $x$

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$2x+1 = A(x^2+1) + (Bx+C)(x-1)$$

$$x=1 \Rightarrow 3 = A(2) \Rightarrow \boxed{A = \frac{3}{2}}$$

EXPANDING  $2x+1 = \frac{3}{2}x^2 + \frac{3}{2} + Bx^2 - Bx + Cx - C = (\frac{3}{2} + B)x^2 + (-B+C)x + (\frac{3}{2} - C)$

↑ equating coefficients ↑

$$\frac{3}{2} + B = 0 \Rightarrow \boxed{B = -\frac{3}{2}}$$

$$-B + C = 2 \Rightarrow \boxed{C = 2 + B = 2 - \frac{3}{2} = \frac{1}{2}}$$

$$\frac{3}{2} - C = 1 \quad \checkmark$$

Extra Practice Problems:

1.  $\int \frac{\sqrt{\ln(x)+5}}{x} dx$

$u = \ln(x) + 5$

$du = \frac{1}{x} dx$

$dx = x du$

$= \int \sqrt{u} du = \int u^{1/2} du$

$= \frac{2}{3} u^{3/2} + C$

$= \boxed{\frac{2}{3} (\ln(x)+5)^{3/2} + C}$

2.  $\int \sin^2(5x) dx$   $\rightarrow$  even powers only  $\Rightarrow$  half angle identity

$= \int \frac{1}{2} (1 - \cos(10x)) dx$

$= \frac{1}{2} \int 1 - \cos(10x) dx$

$= \frac{1}{2} (x - \frac{1}{10} \sin(10x)) + C$

$= \boxed{\frac{1}{2} x - \frac{1}{20} \sin(10x) + C}$

3.  $\int \frac{5}{x^2+2x+10} dx$   $\rightarrow$  doesn't factor  $\Rightarrow$  complete the square

$= \int \frac{5}{x^2+2x+1-1+10} dx = \int \frac{5}{(x+1)^2+9} dx$   $u = x+1$   
 $du = dx$

$= \int \frac{5}{u^2+9} du$   $\leftarrow$  in our table of known integrals!

$= 5 \frac{1}{3} \tan^{-1}(\frac{u}{3}) + C$

$= \boxed{\frac{5}{3} \tan^{-1}(\frac{x+1}{3}) + C}$